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THE BEST FORM OF INDEX NUMBER*

By Irving Fisher

What index numbers are "best?" Naturally much depends on the purpose in view. First let us narrow the question.

Index numbers differ chiefly in: (a) the character and number of data entering into them; (b) the sources and agencies used for collecting the data; (c) the frequency of calculation; (d) the "base" chosen; and (e) the mathematical formula used.

Only the last named, the formula, is here considered.

How widely do index numbers calculated by different formulae differ? Which is (or are) the best?

It occurred to me that new light might be shed on these questions by calculating various index numbers proposed, utilizing the new historical data of the War Industries Board for the years 1913–18.

For this calculation, I have selected thirty-six staple commodities for which complete data, not only for prices, but also for quantities, are given.

The total value of the thirty-six commodities exchanged in 1913 was $13,105,000,000, and in 1918, $29,186,000,000, as obtained by multiplying the prices by the quantities and adding. These figures show an increase of money values in the ratio of 13,105 to 29,186, or of 100 to 223. This 223 per cent is, therefore, the index number of *money value* in 1918 as compared with 1913 as a base.

Our problem is to find how far this ratio of 223 per cent is a matter of inflated prices and how far it is a matter of increased quantities (volume of trade).

It is clear that there can be no ambiguity as to the 223 per cent itself, the index number of money value. There is only one index number of value—the value itself.

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* Professor Fisher's paper, read at the Eighty-second Annual Meeting of the American Statistical Association (Atlantic City, N. J., December, 1920), has not yet been completed in all the detail desirable for publication. Hence only an abstract is printed at this time, together with the discussion that took place at the meeting. The full paper with diagrams and data will be published later.—Editor.

It is only when we try to resolve this 223 per cent into two factors, one the index number of prices and the other the index number of quantities, that we find a bewildering multiplicity of methods. We ought, it would seem, to be able to find a form of index number which, when applied alike to prices and quantities, will give results which check up with the 223 per cent in value.

There are a dozen "tests" which we should like to have an index number fulfil. Eight of these are discussed in my *Purchasing Power of Money.* C. M. Walsh's book, *The Measurement of General Exchange Value,* is chiefly on certain tests of a good index number.

But I have come to the conclusion that there are two supreme tests which, practically at least, carry with them the rest:

I. The formula should *work both ways* as to the two factors, prices and quantities.

II. The formula should *work both ways* as to time.

Thus, as to test I, the formula should, as was said, be such that the two figures it yields when used for calculating the index number of (a) *prices* for 1918 and (b) *quantities* for 1918 will, if multiplied together, give the correct figure, 223 per cent, which represents the index number of *value.*

Again, as to test II, the formula should be such that, if used for calculating, say the index number of prices (a) for 1918 in terms of 1913 as a base and (b) for 1913 in terms of 1918 as a base, the two results will be consistent (*i. e.,* will be reciprocals of each other).

The *simple arithmetic* index number meets neither test.

Thus, the arithmetic index number, when applied to prices, would make it appear that, in 1918, the price level was 187 per cent as compared with 100 per cent for 1913; when applied to quantities it would make it appear that quantities had increased to 126 per cent of what they were in 1913.

But if these two figures are multiplied, we obtain not the true index number of money value of 223 per cent as we should, but 236 per cent, which is too high. The harmonics would give a result too low.

Again, suppose bacon is twice as high in 1918 as 1913 while rubber is the reverse—twice as high in 1913 as in 1918. According to the simple arithmetic index number these two commodities were in 1918 $\frac{200\%+50\%}{2}=125\%$ of what they were in 1913, and at the same time were in 1913 also 125 per cent of what they were in 1918. Each year's price level is 25 per cent above the other's!

In reference to the second test, it may be shown that certain index numbers, including the arithmetic, have an *upward bias,* and others, like the harmonic, a *downward bias.*

It is also true that methods of weighting will introduce a bias. Thus, if the weights used are the values of the *base* year, they impart a *downward bias* to all the index numbers of any given year calculated thereby; while, on the other hand, if the weights used are the values of the given year itself, they impart an *upward bias*.

The reason is, to take the case last mentioned, that the value used in the weighting necessarily contains a relatively high price factor for every high price ratio, and a low for a low. The higher the prices, other things (the quantities) equal, the more they are weighted. It is true that "other things" will not be equal; *i. e.* that the *quantity* element in the weights will have an influence. But this influence is about as likely to be in one direction as the other. This factor, unlike the price factor, is not very definitely correlated with the price ratios which they help weight.

Therefore, the quantity element is of comparatively little consequence in the weights of an index number of prices (or the price element in the weights of an index number of quantities). If I may say so, in an index number of p's we do not need to mind our p's and q's both, but we do need to mind our p's; and vice versa for the index number of q's.

To illustrate, we recur to the bacon and rubber. Suppose that the quantities of bacon are the same in 1913 and in 1918 and the quantities of rubber are likewise the same. It follows that the value of bacon in 1918 was double that in 1913, being proportional to its price and, contrariwise, the value of rubber in 1913 was double that in 1918. To fix our ideas, let us suppose that in 1913 the value of bacon was two million dollars, while in 1918 it was four million dollars, and that the values of rubber were, reversely, three million dollars in 1918 and six million dollars in 1913.

It follows that, with 1913 as a base, the weighted arithmetic index number for 1918, using the values of 1918 (4 for bacon and 3 for rubber) as weights, is

$$\frac{4\times200\%+3\times50\%}{4+3}=136\%$$

while the index number for 1913 on 1918 as base, using the values of 1918 (2 for bacon and 6 for rubber) as weights, is

$$\frac{2\times50\%+6\times200\%}{2+6}=162\%.$$

These weighted arithmetic index numbers are even more absurd than the previously calculated simple arithmetic index number. According to the simple, each year was 25 per cent above the other; but according to the weighted index number just calculated, 1918 is 36 per cent above 1913, and 1913 is 62 per cent above 1918!

Here are two doses of upward bias, one dose being in the nature of the arithmetic average and the other in the way it is weighted. Yet Palgrave proposed just such an index number, not suspecting the double bias involved.

Summarizing: (1) As to bias, it can be shown that the arithmetic average has an upward bias, *i. e.*, has in the very nature of the arithmetic process a natural tendency to give results too large, that the harmonic has a downward bias, that the geometric, median, and mode have no inherent bias in either direction. (2) As to the system of weighting, it can be shown that a system employing the prices of the base year has a downward bias, and one employing the prices of the given year has an upward bias. (3) Consequently, the only arithmetic index number which is free from the faults mentioned is one with weights in which the prices of the base year are used, such as the index number of the United States Bureau of Labor Statistics; for in such an index number the natural upward bias of the arithmetic process is offset by the downward bias in the method of weighting. (4) Similarly, the only harmonic average free from upward or downward bias is that in which the weights contain the prices of the given year. (5) The geometric, median, and mode, having no natural bias, should be doubly weighted so that the prices of both years enter on even terms.

An index number such as that of Professor Day of Harvard, a geometric with weights dependent on prices at or near the base year, will have a downward bias.

What has just been said relates to conformity to test II. There are likewise many formulae which will conform to test I. But only a few formulae conform to both test I and test II. The simplest of these and the "best" by other tests is

$$\sqrt{\frac{p_1q_1+\cdots}{p_0q_1+\cdots}\times\frac{p_1q_0+\cdots}{p_0q_0+\cdots}}.$$

It is a pleasure to learn that Mr. Walsh, approaching the subject from an entirely different point of view, has recently reached the same conclusion. Mr. Walsh's monumental and memorable book, *General Exchange Value*, showed him many years ago to be a master in this field.

While index numbers by different formulae differ materially, index numbers which conform to either one of the two great tests agree remarkably well. This is especially true of the few which conform to *both* tests.

We conclude that index numbers are far more exact than has been realized. Most of the differences are due not to the formula but to the number of commodities, or other conditions mentioned at the beginning, and differences due to differences in formulae almost disappear if we get rid of even the crudest bias.

Properly used index numbers are a precise tool, precise far beyond our needs, precise within much less than 1 per cent.

DISCUSSION

By Wesley C. Mitchell

The index number which Professor Fisher proposes seems to me excellent. Indeed, it is "the best index number" known to me for the particular purpose he has in view. Further, that purpose is an important one, covering as it does most of the uses to which what I call "general-purpose" index numbers are commonly put. But I cannot admit that even perfect adaptation of an index number to any use however important entitles it to be called "the best index number" at large.

This issue is much more than a verbal quibble. So far, most makers of index numbers of prices at wholesale have been designing "general-purpose" series. But the time has come when we are beginning to make a wide variety of index numbers especially adapted to particular uses. And we must go further in that direction. In doing such work the compiler should first define as accurately as may be the use which his new series is to serve, and from this use he should deduce the form of index number which is "the best" for him. This criterion of use should determine the number of commodities to be included, the basis on which commodities are selected, the scheme of weighting, and the type of average. What is best in all these respects for one use may be bad for another use. There cannot conceivably be an index number that is "the best" for all uses.

In fine, I think Professor Fisher has tarnished somewhat his excellent contribution by using incautious language implying universal merits in his formula. His very eminence in this progressive field of work makes that slip important. The path of future progress lies in differentiation. Yet if we took Professor Fisher at his word, we should never produce anything but one admirable kind of "general-purpose" series.

DISCUSSION

By C. M. Walsh

In the first place I wish to make an acknowledgment of appreciation for the flattering remarks with which Professor Fisher has referred to my book. In return I must express my wonder at the stupendous labor he has performed in applying so many complex systems of